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Abstract

In this paper, we study the selection process and incentives of firms that apply for and eventually receive one or multiple governmental grants intended to stimulate innovation and growth in supported firms. The analysis departs from a rent-seeking model of heterogeneous entrepreneurs who are free to allocate their effort between production and rent-seeking. In equilibrium, highly productive entrepreneurs choose not to enter the rent-seeking contest altogether, and moderately productive entrepreneurs allocate a share of their effort both to rent-seeking and production, whereas low-productivity entrepreneurs are incentivized to allocate most, if not all, of their effort to seeking grants and can thus be called *subsidy entrepreneurs*. These firms also have a higher probability of receiving grants. Using detailed data over all grants administered by the three largest grant distributing agencies in Sweden, the empirical analysis suggests that supported firms tend to have relatively low productivity, higher wages, and a larger share of workers with higher education than do non-supported firms. These characteristics become more pronounced as we move from single to multiple supported firms, thus supporting the notion of subsidy entrepreneurs.

Keywords: Rent-seeking · Firm subsidies · R&D grants · Industrial policy

JEL: H25, O38, P16, D72, L52

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1 Introduction

The importance of innovation as a key driver of economic growth can hardly be overstated, and innovation policy is of high priority on the political agenda (Becker, 2015). A recent development in many countries is the extensive use of selective policy instruments, such as R&D grants to increase the innovative capacity of small and medium-sized enterprises (SMEs) in particular. As many SMEs lack sufficient cash-flow and/or collateral to pledge for credit and due to their high risk of failure, acquiring funds from traditional financiers can be difficult. This dearth of funding in the early stages of firm development, together with positive spillover effects from R&D activities (Romer, 1990), provides a hotbed for governments to intervene in capital markets. With the ongoing expansion of R&D subsidies, selective grants now comprise an integral part of the financial ladder for many young start-ups (Tillväxtanalys, 2015). In Sweden, it is estimated that 3.1 percent of the total governmental budget in 2011 was allocated to selective policies targeting innovation-, trade- and industry-related objectives (Tillväxtanalys, 2015).

At the same time, there is increasing concern about the effectiveness of R&D grants and subsidies. In a survey of 77 empirical studies concerning the effect of various public subsidies on firm R&D investment, Zúñiga-Vicente et al. (2014) conclude that the evidence is mixed and inconclusive. Klette et al. (2000) and David et al. (2000) analyze innovation policies, and both suggest that participation in an R&D program is not random. To improve our understanding of the effects from R&D grants and subsidies, they argue that we need to acquire a better understanding of the decisions of both public agencies and firms, thus emphasizing the political economy aspect of selective policies.

Importantly, the objectives of public agents that seek appropriate firms and projects to fund may be at odds with the incentives for firms, leading to less appropriate firms seeking grants. The resulting adverse selection is portrayed by Baldwin and Robert-Nicoud (2007) as a tension in the form of sought *winners* but received *losers*. Although central to our understanding, little attention has been directed to the selection process and the incentives for firms allocating resources to seeking grants. Aerts et al. (2006) even argues that participation equations in empirical work on the impact of various public support programs are typically modeled in an ad-hoc fashion and largely depend on the data available, which is often scant. A further limitation of previous studies on both the selection and effects of subsidies is that they tend to focus on industry rather than firm characteristics.

In this paper, we model and analyze the selection process from the perspective of firms and characterize those that apply for and eventually receive one or multiple governmental grants. To our knowledge, this is the first paper to empirically analyze the determinants of program participation and characteristics of single and multiple supported firms. The analysis is based on unique Swedish firm-level data from the Swedish Agency for Growth Policy Analysis, which cover all grants administered by the three largest grant distributing agencies in Sweden. In total, 15,800 firms received at least one grant during the period 1997-2013, of which 3,600 received two or more grants. These data are linked to detailed firm-level register data from Statistics Sweden, thus making it possible to compare subsidized firms with unsupported firms. One limitation of the data is that we do not know which firms applied for a grant but were denied public funding, nor do we have information about the specific projects presented by the firms for the granting agencies. It is therefore not possible to fully characterize the agencies' screening rules, which is why we focus on the selection of firms into seeking grants and control for program participation to the greatest extent possible.

We argue that Sweden makes a good case for studying the economic implications of selective innovation policy and potential rent-seeking behavior for the following reasons: Sweden is considered one of the most innovative countries in the world, but it is home to a complex system of public innovation programs, with many agencies funding similar things (OECD, 2016). Apparently, the existence of overlapping agencies and programs enhance the formation of rent-seeking firms.

To guide our thinking around selection into rent-seeking, we develop a simple rent-seeking model inspired by Tullock (1980) and Stein (2002). In contrast to previous studies, however, we do not analyze the choice of seeking rents in isolation. Instead, we consider the choice of allocating effort between rent-seeking and productive use (production of a final good). Entrepreneurs are assumed to be heterogeneous with respect to their innate productivity. In equilibrium, we find that entrepreneurs with sufficiently high productivity choose not to enter the grant contest. For entrepreneurs who choose to enter the contest, we find that the amount of effort expended to acquire the grant is inversely related to the level of productivity. Low productivity firms will hence expend more effort in rent-seeking, allocating little if any effort to productive use. With a passive agency administering the contest (assumed throughout our model), these so-called *subsidy entrepreneurs* will also face a higher probability of receiving the grant.

The above argument is based on the opportunity cost of rent-seeking being higher for highly productive firms than for low productivity firms due to forgone profits from the alternative use of effort resources. The results apply to a situation in which all entrepreneurs have the same capacity as rent-seekers. We also consider a situation in which entrepreneurs vary in their capacity as rent-seekers (e.g., in their skill, previous experience, or learning). Then, we find that more rent-seeking skills moderate the inverse relationship between productivity and the effort expended in rent-seeking, thus strengthening the relationship between rent-seeking and low productivity.

In line with the theoretical predictions, we focus the empirical analysis on how single and multiple supported firms deviate from non-supported firms with respect to labor productivity and on how wages and skill-intensity distinguish them from other firms.

In the empirical analysis, we estimate two sets of models. First, we consider a logistic model over the probability to receive a grant. The results suggest that subsidized firms have, on average, lower productivity, higher wages and higher skill-intensity than do non-supported firms. These characteristics are further strengthened when separating single supported firms from multiple supported firms, lending some support to the notion of subsidy entrepreneurs. To complement the binary model, we also consider a multiplicative count model over the number of received grants (1^{st}, 2^{nd}, and 3^{rd} ...) and find similar results, except for the productivity variable, which no longer accounts for selection beyond the 1^{st} count. Nonetheless, we cannot exclude the possibility that a comprehensive system of available grants foster low productive firms to enter into rent-seeking and, to some extent, also specialize in rent-seeking activities.

2 Related literature

The origin of the political economy of rent-seeking can be dated to Tullock (1967) and Posner (1975), and it has since bifurcated in several directions, including strategic rent-seeking (Tullock, 1980), tariffs vs. quotas (Bhagwati and Srinivasan, 1980), risk aversion (Hillman and Katz, 1984), and endogenous rents (Appelbaum and Katz, 1987). An early contribution on the tension between the interest of the government in handing out efficient subsidies and the incentive of firms to seek them dates to Corden (1974), who argued that reductions in real incomes are more unwanted than increases, leading to a bias toward protecting declining industries. Additional examples in this body of literature range from the political support function approach in Hillman (1982) and the tariff-formation function approach in Findlay and Wellisz (1982). A paper by Krueger (1990) described the identity bias leading to the so-called loser's paradox, wherein a large amount of public funds is directed towards ailing rather than expanding industries. We may note that Krueger, as early as 1974, discussed the social cost of rent-seeking in an international trade and developing economies setting (Krueger, 1974). In later years, we have seen a growing number of micro-based lobbying models dealing with governments protecting losers (Magee et al., 1989; Grossman and Helpman, 1994). A recent model is found in (Baldwin and Robert-Nicoud, 2007), who shows that the incentives for lobbying are higher in declining industries with high entry barriers due to sunk costs in capital investments. The suggested mechanism is that in capital-intensive and declining industries, the rents achieved form rent-seeking are less easily competed away by new firms because the high entry cost and bleak outlooks discourage them from entering and free ride on the lobbying efforts done by the incumbent firms.

Empirical evidence for the losers' paradox include Hufbauer and Rosen (1986); Hufbauer et al. (1986), and Ray (1987), who show that declining industries such as agriculture, textiles, and steel receive a disproportionately high level of protection. In addition, Trefler (1993) shows that protection tends to increase with import penetration, whereas Ray (1987) and Hansen (1990) both show that protection tends to be counter-cyclical. On the lobbying connectivity side, Chen et al. (2011) shows that rent-seeking initiatives in China make private firms more prone to establishing political connections and that this mechanism is particularly strong in less market-oriented regions.[1] A related paper in this vein of research is that of Lenway et al. (1996), who show that protection in the US steel sector rewards poor performance, reduces incentives to innovate, and frustrates the normal Schumpeterian process of creative destruction.

An early paper that aimed to model both the impact of and participation in R&D subsidy contests is Takalo et al. (2005), who develops a model that accounts for application costs and isolates the effects of the treatment on the public agency running the treatment program. One result found is that ignoring application costs severely biases the estimated welfare effects of a program. Results on how the selection and screening process may impact program efficiency are provided by Guo et al. (2014), who use Chinese data on the Innovation fund program and find that the effects of program participation increased after 2005, when the project screening became more decentralized. A paper that explicitly focuses on the determinants of participation in publicly sponsored programs is that of Blanes and Busom (2004). The results of their study show that skill-intensive firms with previous R&D experience are over-represented in subsidy programs. When it come to firm size and cash-flow, the results are less clear and suggest that the programs are not picking up the smallest, most inexperienced firms. Finally, there is a tendency to find firms with low cash-flow (which might indicate credit constrained) in these programs.

In an attempt to identify factors that determine program participation, Blanes and Busom (2004) and Afcha (2012) both find that technological cooperation, previous R&D experience, and firm skill-intensity are positively correlated with participation in publicly sponsored R&D incentives. Similar results are obtained by both Cerulli and Poti (2008), who analyzed public R&D subsidies in Italian data, and Czarnitzki and Delanote (2014), who consider German data. In addition, Lööf and Hesmati (2004) find that the firms that received pub-

[1] A survey on rent-seeking and economic development in Asia is given by Khan and Jomo (2000)

lic R&D support could be characterized as R&D-intensive and credit-constrained. These results suggest that aside from the firm characteristics reflecting production structure and productivity, previous experiences in public support programs and innovation behavior may affect the probability of program participation. Along these lines, a report by Feldman and Kelley (2001) on the winners of awards from the Advanced Technology Program in the US found evidence suggesting that the number of business and university linkages a firm had positively affected the probability of winning the contest. In a similar vein, Hussinger (2008) find that the probability of receiving R&D subsidies was positively correlated with having received subsidies previously and with past patenting experience. Similar results pointing at the importance of previous experiences are given by Catozzella and Vivarelli (2011), who analyze innovation subsides using Italian data. They find that innovative productivity was negatively affected by the innovation subsidy and that when being selected into the subsidy, the probability of obtaining the subsidy was higher if the firm had a track record of doing product and process innovations, was export-oriented, and cooperated with universities and/or research institutes.

3 A model of rent-seeking entrepreneurs

To model the selection into seeking grants, we consider a stylized model of rent-seeking entrepreneurs in the tradition of Tullock (1980) and extended by Stein (2002). Of a population of N firms, there are a total of $n \leq N$ firms that are qualified for seeking a particular grant.[2] The value of n is determined by the grant-administering agency and can be seen as a measure of the level of the selectiveness of the grant. As we see below, however, not all of the n qualified firms find it profitable to enter the contest. To model the decision to rent-seek, we thus consider the perspective of an entrepreneurs (or manager) operating one of the n firms. At a given point in time, each of the $i = 1, ..., n$ entrepreneurs dispose of a unit measure of effort that can be allocated either to the production of a final good or to seeking the grant. We consider $h_i \in [0, 1]$ to be the effort allocated to rent-seeking, including writing of applications, lobbying activities, and gathering the information required for the firm to comply with the guidelines set up by the governmental agency handing out the subsidy. The remaining effort is given by $1 - h_i$ and is allocated to productive effort. If $h_i = 1$, all effort is spent on rent-seeking, and if $h_i = 0$, all effort goes into productive use. Expending effort comes at no costs with no possibility of saving, which means that entrepreneurs will expend all their effort. In the production of the final good, effort is augmented with an entrepreneur-specific and exogenous productivity term A_i. With a market price of the final good given by p, the

[2] Here, we use the terms rents and grants interchangeably

profit from productive effort earned by entrepreneur j is described as follows:

$$\Pi_j^P(h_j) = pA_j(1 - h_j). \tag{1}$$

There are no conditions such as fatigue that could cause the profit to be decreasing for expending additional effort, nor does the entrepreneur employ other factor in production of the good. Of course, these are gross simplifications, but they are plausible for new ventures, at least in the short run. Many start-ups have no employees and little physical capital, but if they do, these factors are likely to be more or less fixed.[3] Given the productivity term A_i that embodies entrepreneurial human capital, the incentives for allocating effort to production depend on the expected profits that could be earned from allocating effort to rent-seeking instead of production.

The incentives for seeking grants or subsidies is traditionally modeled using some form of contest function, where contestants expend a certain amount of effort to compete for a monetary price, here given by μ. In the spirit of Tullock (1980), we consider the following setup, where the probability for a risk-neutral entrepreneur j of winning the contest and acquiring the grant is given by $P_j(\boldsymbol{h}) = h_j / \sum_{i=1}^n h_i$. The probability of winning is increasing with h_j and decreases in the efforts expended by the other contestants through $\sum_{i=1}^n h_i$. Throughout the model, we make the standard assumption that the governmental agency administering the contest is passive, in the sense that the grant is handed out randomly among the entrepreneurs entering the contest, with no regard to the distribution of A_i. This assumption, however, does not prohibit the distribution of A_i from factoring into the decision in limiting the number of contestants to n. For the j entrepreneur, the expected profits from rent-seeking can be described by the following expression:

$$\Pi_j^R(\boldsymbol{h}) = \mu \frac{h_j}{\sum_{i=1}^n h_i}. \tag{2}$$

While the productive capacity differs between entrepreneurs, they share the same constant returns of rent-seeking technologies. With complete information about the distribution of A, the problem faced by each entrepreneur is to determine how much effort to expend on production and rent-seeking. For entrepreneur j, it means finding the level of effort $0 \leq h_j^* \leq 1$ that maximizes the expected profits of both $\Pi_j^P(h_j)$ and $\Pi_j^R(\boldsymbol{h})$ using the

[3] In this setting, additional factors would only affect the output (profits) by shifting the productivity term A_i. To keep the model analytically tractable, we dispense with additional structure.

following objective function:

$$\max_{h_j} \quad \Pi_j(\boldsymbol{h}) = A_j(1-h_j) + \mu' \frac{h_j}{\sum_{i=1}^n h_i}, \tag{3}$$

where the price of the final good has been normalized to one and the rent-seeking price is expressed as the relative price of $\mu' = \mu/p$.[4]

To solve the model, differentiating (3) with respect to h_j results in n separate first-order conditions. For the j-th entrepreneur, it means that the marginal revenue of the rent-seeking effort equals the marginal revenue of productive effort with

$$\mu' \frac{\sum_{i=1}^n h_i - h_j}{\left(\sum_{i=1}^n h_i\right)^2} = A_j. \tag{4}$$

Taking the second-order derivative of the objective function, we see that

$$\frac{d^2\Pi_j(\boldsymbol{h})}{dh_j^2} = -2\mu' \frac{\sum_{i=1}^n h_i \left(\sum_{i=1}^n h_i - h_j\right)}{\left(\sum_{i=1}^n h_i\right)^4} < 0, \tag{5}$$

for all values of $h_i \in [0,1]$, thus showing the existence of a global maximum. In case entrepreneurs are identical in their productive effort, i.e., setting $A_i = A_j$ for all i, the solution is the textbook symmetric equilibrium, with two exceptions. First, unless $A < \mu'(n-1)/n^2$, no entrepreneurs will enter the contest, resulting in $h_j^* = 0$, and second, after entering the contest, $h_j^* = \max\{\mu'(n-1)/An^2, 1\}$, considering the possibility of a corner solution. The allocation of effort to productive activity therefore becomes $1 - h^* = \max\{1 - \mu'(n-1)/An^2, 0\}$. However, with different productivity endowments, we now turn to the characterization of an asymmetric interior equilibrium solution with $k \leq n$ entrepreneurs who choose to enter the model. Following Stein (2002), who studies a similar contest, we proceed by ordering the entrepreneurs in ascending order according to their A_i, such that

[4] In this setting, all contestants operate on the same market and produce the same good with a single price. The model can be extended to one in which the contestants produce heterogeneous goods for different markets. In this case, the relative price becomes entrepreneur-specific, i.e., μ_j for the j-th entrepreneur. One interpretation of μ_i is pursued by Stein (2002), with contestants having different *valuations* of the price. Solving the model with μ_j instead of μ, however, does not alter the solution fundamentally. Another possible extension is to consider a so-called *matching grant*, where the winner of the contest is required to match the received grant with their own funds (Klette and Møen, 2012). One way to account for this is to add a fixed-cost term $-s\mu' P_j(\boldsymbol{h})$ to the expression in (2) that is paid in case of winning, where $s\mu'$ is the fraction of the relative price that the winner must match. The inclusion of such a fixed cost does not alter the solution in any significant way, but it does lower the prize to $\mu'(1-s)$ and thus decreases the overall rent-dissipation.

$0 < A_1 \leq A_2 \leq \ldots \leq A_n$. Solving for h_j in (3) we obtain

$$h_j = \sum_{i=1}^{n} h_i - \frac{A_j}{\mu'} \left(\sum_{i=1}^{n} h_i \right)^2 . \tag{6}$$

From the slackness condition of maximizing (3), it follows that $\Pi'_j(h_j) = 0$ if $0 < h_j \leq 1$ and $\Pi'_j(h_j) \leq 0$ if $0 = h_j$. Using this fact together with the ordering of A_i, we see from (6) that for a sufficiently large A_i, $h_j = 0$. This means that there exists some index k such that for the $i = 1, ..., k$, entrepreneurs, $1 \geq h_1 \geq h_2 \geq \ldots \geq h_k > 0$ and for the remaining $i = k+1, ..., n$ entrepreneurs, $h_{k+1} = h_{k+2} = \ldots = h_n = 0$. These high productive entrepreneurs abstain from entering the contest altogether and allocate their total effort to productive use. To solve the model for the $i = 1, ..., k$ entrepreneurs, we proceed by taking the sum over the k first firms in (6). Solving for $\sum_{i=1}^{k} h_i$, we get

$$\sum_{i=1}^{k} h_i = \mu' \frac{k-1}{k\bar{A}_k}, \tag{7}$$

where $\bar{A}_k$ is the arithmetic average of the k first (low) productivity terms. This expression shows the aggregate effort allocated to rent seeking and can be used together with (6) to find the profit maximizing h_j^* for the j-th entrepreneur. After simplifying terms, we obtain

$$h_j^* = \max \left\{ \mu' \frac{k-1}{k\bar{A}_k} \left(1 - \frac{(k-1) A_j}{k\bar{A}_k} \right), 1 \right\}, \tag{8}$$

where the max operator comes from the possibility of corner solutions (a firm cannot allocate more than 1 to rent seeking), which is a consequence of h_j being defined on the unit interval. The optimal allocation of effort to productive use can likewise be determined by

$$1 - h_j^* = 1 - \mu' \frac{k-1}{k\bar{A}_k} \left(1 - \frac{(k-1) A_j}{k\bar{A}_k} \right), \tag{9}$$

which, by construction, satisfies $0 \leq 1 - h_j^* < 1$, which follows from the participating constraint of $0 < h_j$ and the fact that $h_j^* \leq 1$ in (8). Note that by dividing (6) with (7), we obtain

$$P_j \left(h_j^* \right) = 1 - \frac{(k-1) A_j}{k\bar{A}_k}, \tag{10}$$

which is the probability P_j that entrepreneur j wins the contest. Since P_j is a probability, $1 \geq P_j \geq 0$. Summing over all k entrepreneurs in (10), it follows that $\sum_{i=1}^{k} P_j = 1$. In equilibrium, it follows from (10) that $A_i[1 - P_j(h_j^*)] = A_j[1 - P_i(h_i^*)]$, for all contestants $i \neq j$. Any difference in productivity between two contestants is fully compensated for by the probability of losing the contest. Moreover, for the k participating entrepreneurs, the expected profit from rent-seeking should be positive. This can be verified by inserting the expressions (8), (9), and (10) into the objective function in (3), yielding

$$\Pi_j\left(h_j^*\right) = A_j + \mu'\left(1 - \frac{(k-1)\,A_j}{k\bar{A}_k}\right)^2 > 0. \tag{11}$$

Similar to the result in *Proposition 1*, as given in Stein (2002), we can from either (10) or (8) find the index value k that determines the last entrepreneur (in the ascending order defined previously) who finds it profitable to enter the contest and therefore have $h_k > 0$ and $P\left(h_j^*\right) > 0$. We present this result of a participation constraint in the following proposition.

Proposition 1 (Participation constraint)**.** *The last entrepreneur who enters the rent-seeking contest with index $k \geq 2$ has a productivity parameter A_k that is sufficiently small to satisfy the following inequality:*

$$A_k\,(k-2) < \bar{A}_{k-1}\,(k-1)\,, \tag{12}$$

where $\bar{A}_{k-1}$ is the arithmetic average of the $k-1$ preceding (smaller) A_i-terms.

Proof. Since $P_1 \geq P_2 \geq \ldots \geq P_k > 0$ if an only if $h_k > 0$, together with $h_{k+1} = 0$, we have that $P_{k+1} = 0$. From either (10) or (8), it must therefore be true that

$$P_{k+1} = 1 - \frac{(k-1)\,A_{k+1}}{k\bar{A}_k} = 0,. \tag{13}$$

Mapping $k \mapsto k-1$ in (13), it follows that k is the largest index that satisfies the inequality

$$P_k = 1 - \frac{(k-2)\,A_k}{(k-1)\,\bar{A}_{k-1}} > 0, \tag{14}$$

which after solving for A_k yields the desired inequality for $k \geq 2$. □

Thus, while the entrepreneurs who do not enter the contest and allocate all their effort to to production earn a profit of A_i, the expected profits for entrepreneurs who enter the contest but lose differ from the realized profits of $A_j\left(1 - h_j^*\right)$, which satisfies $A_j > A_i\,(1 - h_i^*) \geq 0$.

Returning to the optimal allocations h_j^* and $1 - h_j^*$ arrived at in (8) and (9), we can illustrate the allocation between rent-seeking and production in Figure 1, which plots the marginal profit from both rent-seeking and production. Beginning with rent-seeking (dashed line), it shows the marginal profits (left vertical axis) for different h_j (right on the horizontal axis). As the entrepreneur allocates more effort to rent-seeking, the marginal profit decreases. In the case of an interior solution, this line will, at some point, cross the constant (drawn) line between 0 and 1 that gives the marginal profits from allocating effort to production (right vertical axis). To maximize profits, the entrepreneur stops rent-seeking at that point and instead turns the remaining effort to productive use (right on the horizontal axis). In the figure, the optimal allocation to rent-seeking is given by $h_j^* = 0.42$, and that given to productive use is $1 - h_j^* = 0.58$. Because of the participation constraint, none of the k entrepreneurs who enter the contest face a marginal profit line from production, which lies strictly above the profit line for rent-seeking because $h_j > 0$. However, it is possible that some or all entrepreneurs face a profit line from production that lies strictly below the profit line from rent seeking. These entrepreneurs face a corner solution and will use all their effort in rent-seeking. This figure also anticipates a number of results of what happens when

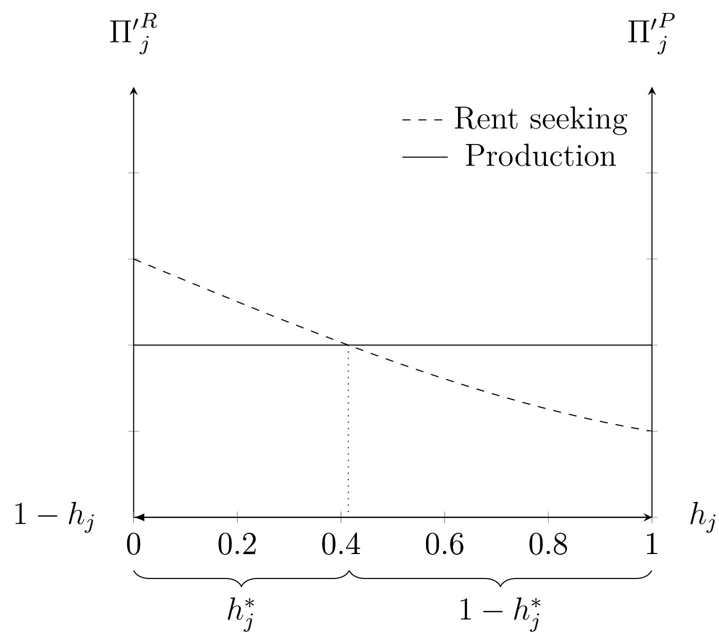

Figure 1: The allocation of entrepreneurial effort between productive activity and rent-seeking

considering the comparative statics in the model. We begin by looking at what happens to the probability of winning the contest when the entrepreneur's own productivity changes and when another entrepreneur experience a shock to his or her productivity. We collect the results in the following proposition

Proposition 2 (Comparative statics I). *For the j-th entrepreneur and with $2 \leq j \leq k < n$ and k, n held fixed, we find that an increase in productivity A_j leads to* *(i) A decrease in the winning probability of entrepreneur j:*

$$\frac{dP_j\left(h_j^*\right)}{dA_j} = -\frac{1}{k\bar{A}_k}\left((k-1) - \frac{(k-1)\,A_j}{k\bar{A}_k}\right) < 0 \tag{15}$$

(ii) An increase in the winning probabilities of all other entrepreneurs $i \neq j$:

$$\frac{dP_i\left(h_i^*\right)}{dA_j} = \frac{(k-1)\,A_i}{\left(k\bar{A}_k\right)^2} > 0. \tag{16}$$

Proof. The inequality in (i) follows directly from the conditions guaranteeing that $P_j(\boldsymbol{h}) > 0$ provided $k \geq 2$. In the case of (ii), the results follow directly from the definitions of the parameters. □

The result in (15), together with the participation constraint in (12) predicts an overall inverse relationship between the probability of receiving a grant and productivity. We also see that the size of the grant μ' has no effect on the probability of winning the contest. Another implication is that although the total welfare decreases for higher k as more resources are allocated away from productive use, the welfare loss is moderate (particularly for small k) because it is mainly low-productivity entrepreneurs who expend more effort in rent-seeking. In Appendix A.1, we provide additional results for how profits Π_i^* and allocated effort h_i^* respond to changes in A_i.

To illustrate the relationship among A_i, k, h_i^*, $1-h_i^*$, $P_i(win)$, and Π_i^*, we give the following numerical example, where the distribution of productivity is given by $A_{i+1} = A_i + \Delta$, with $A_1 = 1$ and $\Delta = 0.1$. With this configuration, we can solve for the participation constraint analytically, Proposition 6 in Appendix A.1 shows that $k < 6$. If the monetary (relative) price is set to $\mu = 1$, the equilibrium solution can be characterized in the following Table 1. Indeed, we see that only five firms will expend a positive h_i to earn an expected

Table 1: Example with productivity parameters A_i being equidistantly distributed

Firm (i)	A_i	Find $k > 2$	h_i^*	$1-h_i^*$	$P_i^*(win)$	Π_i^*
1	1		0.222	0.778	0.333	1.111
2	1.1		0.178	0.822	0.267	1.171
3	1.2	2.100	0.133	0.867	0.200	1.240
4	1.3	1.650	0.089	0.911	0.133	1.318
$5 = k$	1.4	1.533	0.044	0.956	0.067	1.404
6	1.5	1.500	0.000	(1.000)	0.000	(1.500)
Total			0.666	4.334	1	6.744

profit with a positive probability. In the example, the sixth entrepreneur with productivity given by $A_6 = 1.5$ does not find it profitable to enter the contest and will therefore expend all effort in production. The total expenditures (or rent-seeking dissipation) amount to 0.666, which is less than full rent dissipation. Hence, the value of the reward exceeds total expenditures on rent seeking, suggesting that the grant is a net subsidy to competing firms. Moreover, from Proposition 5 in Appendix A.1, it follows that if one entrepreneur experiences an increase in productivity, A_i, all other (competing) entrepreneurs will increase their rent-seeking expenditures h_i because $P_i(\boldsymbol{h}^*) < 1/2$ for all $i \leq k$. In the above situation with unit price μ', the social impact of rent-dissipation can be seen directly as it corresponds to the *loss in productive output* (LPO), here given by $LPO = \sum_{i=1}^{k} A_i(1 - h_i^*)/\sum_{i=1}^{k} A_i$. Using (1) and (7), we can define the LPO as

$$LPO = \frac{1}{\bar{A}_k}\left(1 - \frac{\mu(k-1)}{k^2\bar{A}_k}\right). \tag{17}$$

In the example, it is given by 0.874, which means that the productive output is 12.6% lower under rent-seeking among the 5 entrepreneurs than it is in the absence of rent-seeking.

Although simple, this model provides a basic framework for studying the selection process in grant-seeking from the perspective of the entrepreneur. In short, it shows that low-productivity entrepreneurs are incentivized to seek grants, which makes them expend more effort that, in turn, increases their probability of winning compared to more productive entrepreneurs. In the empirical analysis, we distinguish between firms that are observed to receive one grant from firms that receive more than one grant. Although we do not explicitly model how entrepreneurs adapt to a setting with repeated contests, we try to address this question in the present framework by generalizing the model in next section.

3.1 Heterogeneity in rent-seeking capacity

So far, entrepreneurs only differ with regards to their entrepreneurial productive capacity, i.e., the parameter A_i, while rent-seeking ability is the same for all contestants. We hereby lift this assumption and introduce a parameter $\theta_i > 0$ that captures the differences between entrepreneurs in their capacities as rent-seekers. It captures the administrative efficacy in filling out forms, the capacity of lobbying and the ability to adjust the relevant projects to meet specific requirements set by the responsible agency. Entrepreneurs with high θ_i have acquired skills that make them more efficient as rent-seekers compared to firms with lower θ_i. Although θ_i is given exogenously, we can envision that the acquisition of skills, at least in part, comes from previous experiences in rent-seeking (i.e., from seeking previous grants). To relate θ_i to learning, we could let $\theta_i = \alpha h_i^*$, where $0 \leq h_i^* \leq 1$ is the effort expended on rent seeking from a previous contest and α is the degree of learning. An alternative interpretation of θ_i is that it represents the quality of the project that the entrepreneur is trying to finance with the grant. Seeking a grant with a low-quality project would then require more effort than that with a high-quality project to maintain the same probability of being awarded the grant. Although the grant-seeking agency is assumed to hand out the grant at random, this interpretation of θ_i acknowledges the desire of public officials to award grants to high-quality projects. Regardless of the interpretation, the objective function is written as

$$\max_{h_j} \quad \Pi_j(\boldsymbol{h}) = A_j(1-h_j) + \mu' \frac{\theta_j h_j}{\sum_{i=1}^n \theta_i h_i}. \tag{18}$$

The rent-seeking technology is akin to the one in Stein (2002), referred to as *different relative effectiveness*, although in a different setting. The first-order condition for the j-th entrepreneur can now be expressed as

$$\theta_j h_j = \sum_{i=1}^n h_i - \frac{A_j}{\mu' \theta_j} \left(\sum_{i=1}^n h_i \right)^2. \tag{19}$$

Let $\gamma_i := A_i/\theta_i$ and order entrepreneurs by their γ_i, such that $0 < \gamma_1 \leq \gamma_2 \leq \ldots \leq \gamma_n$. It then follows from (19) that $\theta_1 h_1 \geq \theta_2 h_2 \geq \ldots \geq \theta_n h_n$. As before, we can find an equilibrium solution with $k < n$ entrepreneurs who find it profitable to enter the rent-seeking contest.[5] Proceeding as in the previous section, the equilibrium solution for the j-th entrepreneur is given by

$$\theta_j h_j^* = \max\left\{ \mu' \frac{k-1}{k\bar{\gamma}_k} \left(1 - \frac{(k-1)\gamma_j}{k\bar{\gamma}_k}\right), \theta_j \right\}, \tag{20}$$

where $\bar{\gamma}_k$ is the arithmetic average over the k first γ_i terms.[6] This expression is similar to the expression in (8), where the probability of receiving a grant becomes

$$P_j^* = \left(1 - \frac{(k-1)\gamma_j}{k\bar{\gamma}_k}\right). \tag{21}$$

The probability of winning the grant is still negatively related to productivity A_j, but it is moderated by the entrepreneur's capacity θ_j as a rent-seeker. We formalize this result in the following proposition:

Proposition 3 (Comparative statics II). *For the j-th entrepreneur and with $2 \leq j \leq k < n$*

[5]There is no *ex ante* reason why the entrepreneurs $i = 1, ..., k$ are the same as $i = 1, ..., k$ defined in the model with no heterogeneity in rent-seeking capacity.

[6]Should, e.g., the ability to rent-seek perfectly offset the differences in productivity between firms such that $A_i/\theta_i = A_j/\theta_j$ for all firms $i \neq j$, the solution reduces to $h_j^* = \frac{\mu'(k-1)}{A_j k^2}$.

and k, n held fixed, then

(i) An increase in productivity A_j leads to a decrease in the winning probability of entrepreneur j:

$$\frac{dP_j^*}{dA_j} = -\frac{(k-1)^2\,\bar{\gamma}_{i\neq j}}{\theta_j\,(k\bar{\gamma}_k)^2} < 0, \tag{22}$$

where $\bar{\gamma}_{i\neq j}$ is the arithmetic average over the $(k-1)$ numbers of γ_i, $\forall i \neq j$, and $i \leq k$.

(ii) An increase in rent-seeking capacity θ_j leads to an increase in the winning probability of entrepreneur j:

$$\frac{dP_j^*}{d\theta_j} = \frac{(k-1)^2\,A_j\bar{\gamma}_{i\neq j}}{(k\bar{\gamma}_k)^2} > 0 \tag{23}$$

(iii) The decrease in winning probability from an increase in A_j is positively moderated by an increase in θ_j for entrepreneur j iff $k \geq 3$:

$$\frac{d^2P_j^*}{dA_j d\theta_j} = \frac{(k-1)^2\,\bar{\gamma}_{i\neq j}}{(\theta_j k\bar{\gamma}_k)^2}\left(1 - \frac{2\gamma_j}{k\bar{\gamma}_k}\right) > 0, \quad \text{if } k \geq 3, \tag{24}$$

(iv) The decrease in winning probability from an increase in A_j for entrepreneur j is negatively moderated by an increase in the θ_i of another entrepreneur $i \neq j$:

$$\frac{d^2P_j^*}{dA_j d\theta_i} = -\frac{(k-1)\,\gamma_i}{\theta_j\theta_i\,(k\bar{\gamma}_k)^2} < 0 \tag{25}$$

Proof. Here *(i)*,*(ii)*, and *(iv)* follow directly from the definition of the parameters. In the case of *(iii)*, it suffices to consider $1 > P\left(h_j^*\right) > 0$, provided $k \geq 3$. □

The result in (i) is analogous to that in *(i)* of Proposition 3. From *(ii)*, we see that entrepreneurs who have more experience (higher θ_i) have a greater probability of winning the contest. In *(iii)*, we see that rent-seeking skills moderate the decreased winning probability that results from higher A_j. In small contests, however, with $k = 2$, this need not be the case. This result predicts that the productivity of entrepreneurs who have acquired more rent-seeking skills (e.g., through previous rent-seeking experience) would be less negatively associated with the probability of winning the contest compared to that of entrepreneurs who have acquired less skills. The reverse is true for an increase in θ_i, as shown in *(iv)*. Thus, if another entrepreneur i acquires more rent-seeking skills, it further decreases the winning probability for entrepreneur j, which results from higher A_j.[7]

[7]We can also solve for the index k using the modified inequality constraint for the highest k given by

Before turning to the empirical method, we discuss some limitations of the model and how they relate to the choice of entering many contests. First, there is no uncertainty in our models. Entrepreneurs have complete information about the distributions of both A_i and θ_i. In real contests there may, for example, be an unexpected element to seeking grants if entrepreneurs are uncertain about their own rent-seeking capacity, i.e., θ_i, that they learn only after entering a contest. Although we do not model it explicitly, we can sketch a situation in which entrepreneurs are ignorant about their own and others' θ_i, acting as though $\theta_i = 1$, for all $i \leq k$. In this case, a number of entrepreneurs will over-invest in rent-seeking, while others will under-invest. Entrepreneurs who over-invest may, e.g., suffer unexpected losses in profits, making them perhaps more reluctant to engage in future rent-seeking contests, or if they do engage, they reduce their effort accordingly.

In the data, we observe that most firms stop seeking subsides after one round, which may be partly due to entrepreneurs overestimating their own capacity as rent-seekers. This tendency is well documented by, e.g., Forbes (2005), who finds a general tendency of entrepreneurs to be over-confident. A second deterrent from seeking many grants is the unexpected conditions that may follow from winning the grant, as suggested by the following anecdotal evidence from one entrepreneur who received 300 000 SEK from Vinnova (approximately 30 000 Euro). He complained that he was required to spend money on other things than what he himself would have liked, which, in his opinion, made the subsidy inefficient.[8] Any one of these possibilities may factor into the actual choice of seeking grants, but it lies somewhat outside of our primary interest in assessing the role of productivity among firms that select to rent-seek. Since we do not observe rent-seeking capacity from our data, the empirical analysis focuses primarily on the results derived for productivity.

We conclude the theoretical discussion by summarizing our findings in the form of a combined hypothesis that we subject to empirical testing. Here, we shift the focus from entrepreneurs to firms and hypothesize that (1) low-productivity firms are more likely to apply for a grant and (2) given a number of firms that apply for a grant, the low-productivity entrepreneurs are also more likely to receive the grant. In the first case, our theory predicts that low-productivity firms have lower opportunity costs in allocating effort to writing applications, lobbying, and other such actions than do more productive firms (*Proposition* 1). In the second case, having applied for a grant, low-productivity firms are predicted to expend more effort than are more productive firms (*Proposition* 2 (i)), which increases the probability of receiving the grant.

$(k-2)\,\gamma_k < (k-1)\,\bar{\gamma}_{k-1}$. In this case, however, we cannot say anything directly about the size of either A_k or θ_k, which determines whether an entrepreneur is willing to enter the contest because it is only their ratio γ_i that is relevant.

[8] See story (in Swedish) at Sveriges Radio.

4 Data and description

To analyze the selection process of firms seeking and eventually receiving grants, we use data from a unique database provided by the Swedish Agency for Growth Policy Analysis, which collects information on grants from several agencies in Sweden. It includes information on the size and timing of the grant that we use to construct our dependent variables in the analysis (see the following section on empirical method). The data also include a unique firm identifier that enables us to match the grant data with an already-matched employer-employee data set. The information on grants covers the period between 1997 and 2013 and includes grants from the Swedish Innovation Agency (Vinnova), Swedish Energy Agency (SEA) and Swedish Agency for Economic and Regional Growth (SAERG). These are the three largest grant administering agencies in Sweden, which means that our sample covers a large share of all selective grants given to firms during the period. In short, SAERG promotes entrepreneurship in general and provides EU-based regional subsidies, while Vinnova focuses on projects related to innovation and R&D that could be classified as more high-risk, whereas the SEA focuses on projects in the energy sector. Although the agencies are, to some extent, overlapping and fund similar things (OECD, 2016), e.g., SEA having innovation grants to firms in the energy sector, we can use the separate profiles of the agencies to form a broad typology of grants, referring to the grants given by Vinnova as *innovation* grants, by SAERG as *regional* grants, and by SEA as *energy* grants. In Table 2, we present a summary of the distribution of grants by type, distinguishing between the number of grants that go to a single firm (single grants) and grants that go to more than one firm (multiple grants). In the

Table 2: Descriptive statistics over grants and subsidized firms

Type of grants	Innovation (1)	Regional (2)	Energy (3)	Total (4)
Number of single grants	1,834	10,229	181	12,244
Number of multiple grants				
Innovation	654	587	195	1436
Regional		2,100	38	2,138
Energy			24	24
Total	2,488	12,916	438	15,842

table, we see that the SAERG is responsible for the largest share of both single and multiple grants, followed by Vinnova and SEA. In total, 15,842 Swedish firms received at least one subsidy some time in the period, of which 3,598 firms were granted two or more grants. In the main analysis, we choose to combine the data from each agency. However, because the specifics of each grant differ in the criteria for which firms and projects are qualified to apply,

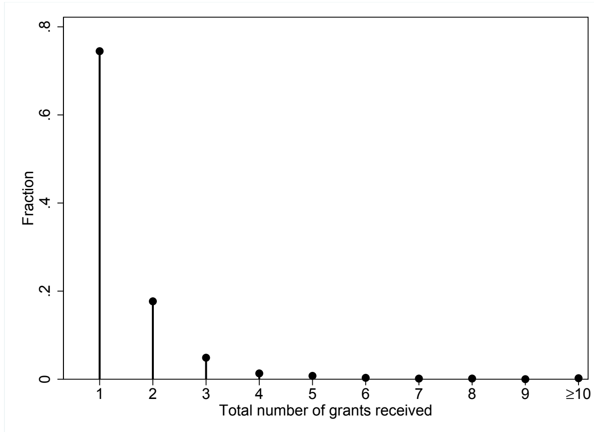

Figure 2: Fraction of firms that receive n number of grants in the observed period

in the size of the grant and in the purpose of the grant, we also run separate analyses using grants for each agency. In Figure 2, we present a frequency plot over the total number of grants a firm recieves during the period. Clearly, most firms receive either one or two grants, but a handful of firms receive more than 10 grants. The highest number of grants received by a single firm is 38.

The firm-level data come from Statistics Sweden (SCB) and provide us with variables such as the sales, value added, investments, capital stock, equity and educational attainment of the labor force for all firms in Sweden, regardless of their size and legal status.

In the ensuing analysis, our ambition is to extract characteristics common to firms that engage in the grant-seeking process. To accomplish this, we also need information about the firms that did not select to seek a grant. However, the set of non-supported firms is several magnitudes larger than the set of subsidized firms and contains a great deal of noise that may obfuscate the analysis. To deal with some of this noise, we impose a number a number of limitations. First, it is well known that the smallest firms are extremely volatile, and after looking at the data, we find that many of these firms lack information on several of the variables used in the analysis. Still, as many grant-seeking firms are small and of relatively young age, we do not want to exclude too many, lest we risk skewing the size distribution of non-supported firms beyond that of the subsidized firms. We have therefore chosen a conservative cut-off and only drop firms with zero or one employee, regardless of their level of sales. Second, the analyzed grants are typically targeting private service and manufacturing firms that are not active in the primary or public sectors. Therefore, firms with a NACE-code related to agriculture, restaurants and publicly funded industries were excluded[9].

[9] More specifically, firms related to NACE (rev.1) codes 1-5, 55 and 75-99 were excluded.

Our main focus is how productivity at the firm level relates to the process of receiving grants. As we do not observe entrepreneurial productivity in the data, as suggested by the theoretical model, we need to find a proxy for this variable. Here, we use (log) labor productivity, as calculated by the value added per employee (*Labor productivity*).[10] Although it does not capture the aspects of productivity that come from physical capital in production, labor productivity is relatively simple to calculate. As a robustness test, we have also considered the total factor productivity, which yielded similar results. In addition to labor productivity, each of the econometric models are fitted using (log) average labor cost (*Labor cost*) in place of the labor productivity variable, representing the cost side in the firm. This accomplishes two things: first and foremost, it gives an alternative measure of labor productivity and can thus be seen as a robustness test. Second, under the assumption that workers are paid according to their marginal product, the resulting estimates should coincide. Hence, by studying both variables, we can assess the extent to which there is a discrepancy between wages and marginal product of labor. For example, if the resulting estimates of labor productivity were more negative than were those of wage cost, we would suspect that rent-seeking firms, while having low productivity, pay wages below their marginal product, possibly to boost profits. Subtracting the average labor cost from the average labor productivity gives us a measure of firm profit in the form of the gross operating surplus (Vandenberghe, 2013). It represents the amount of funds remaining once labor expenses have been paid, which can used to pay debtors and make investments.[11] For this reason, we chose not to enter both variables in the same regression, which would alter the interpretation of labor productivity to that of the gross operating surplus. In both models, we use additional variables to control for the remaining differences between subsidized and non-subsidized firms that could factor into the selection process. These are the (log) number of employees (*Nr. employees*), the ratio of highly skilled laborers to total employment (*Share high skill*), and the firm's equity ratio (*Equity ratio*), together with a set of industry (2-digit), regional controls (NUTS2), and year dummies. We expect that larger firms will be more likely to receive subsidies because there might be benefits of scale when it comes to the application process. For example, a larger firm might have one dedicated employee to fill out applications, which a smaller firm could not afford. The equity ratio of the firm is a measurement of the firm's financial situation. The lower the equity ratio is, the more indebted the firm is, which might indicate financial difficulties. The skill intensity of the firm controls for human capital in the firm. In addition, skill-intensive firms with a relatively high administrative capacity might be better

[10] We present the variable label emphasized in brackets following each of the explanatory variables. This label is then used in the summary and result tables

[11] Definition from the OECD.

equipped to handle this type of task than are other firms.

In Table 3, we present descriptive statistics for all of the variables included in the empirical analysis.[12] Note that we here only consider observations from before receiving a grant because we are interested in describing the firms that select to seek grants as opposed to the firms that have received a grant.

Table 3: Summary statistics over explanatory variables

	Observations	Mean	Median	Std. Dev.	Min	Max
Never subsidized						
Nr. employees (log)	2,127,977	1.6	1	0.971	0.693	9.507
Labor cost (log)	2,118,519	5.3	5	0.596	-3.69	11.154
Share high skill (log)	1,992,312	23	6	31.2	0	100
Equity ratio	2,125,398	0.21	0	11.4	-11999.006	648.043
Labor productivity (log)	2,081,936	5.9	6	0.694	-6.692	13.780
Single subsidized						
Nr. employees (log)	34,718	2.3	2	1.45	.693	10.057
Labor cost (log)	34,621	5.4	5	.516	-1.245	9.391
Share high skill (log)	33,747	29	17	31.9	0	100
Equity ratio	34,715	.28	0	.359	-19.575	14.349
Labor productivity (log)	33,633	6	6	.642	-.929	11.043
Multiple subsidized						
Nr. employees (log)	9,228	3.1	3	1.97	.693	10.424
Labor cost (log)	9,205	5.6	6	0.496	-1.08	9.412
Share high skill (log)	9,051	41	32	34.6	0	100
Equity ratio	9,227	0.3	0	0.418	-31.517	0.999
Labor productivity (log)	8,677	6.1	6	0.656	-1.026	10.745

Notes: Summary statistics for subsidized and non-subsidized firms from 1997-2013. The statistics for the subsidized firms are before they receive their subsidies.

In the table, we see that the number of observations for never subsidized firms far exceeds those of single and multiple supported firms. This is to be expected because a total of 15,842 firms received at least one grant at some time during the period. Since we are interested in the selection process of firms leading up to them receiving a grant, observations are only included prior to receiving a grant in the table. For multiple supported firms, this essentially means observations until their last grant. However, to limit the effect of preceding grants for multiple supported firms, observations obtained between grants are not included in the table. Looking at the data for our key variables *labor productivity* and *labor cost*, we note that labor productivity, on average, increases slightly as we move from never subsidized firms (5.9) to single subsidized (6) and multiple subsidized (6.1). For labor costs, we observe a

[12]Descriptive statistics for firms but divided by type of grant (agency) are available by request from the authors

similar trend of 5.3, 5.4 and 5.6 for the respective categories of firms. These observations go against our hypothesis of labor productivity, being negatively associated with receiving an additional grant. However, these figures are unconditional, so to test the hypotheses, we formulate an econometric model in the next section. Before we turn to the empirical method, we make the following comments regarding our additional control variables. We see that multiple subsidized firms are larger, with 3.1 employees on average, compared to 2.3 and 1.6 for single subsidized and never subsidized, respectively, which could account for some of the trends observed for labor productivity and labor cost. Finally, we observe the highest equity ratio and share of workers with a post-secondary education among the multiple subsidized firms.

5 Empirical method

To analyze the selection process into seeking and receiving grants, we estimate a number of discrete regression models. We begin by estimating a binary model of the probability of receiving a grant, conditional on firm level and control variables $X_{i,t-1}$, i.e., $P\left(T_{it}=1|X_{i,t-1}\right)$, where T_{it} takes the value 1 if a firm i receives a grant at time t, and 0 otherwise. In this model, we do not distinguish between the number of grants a firm receives. To begin, we only look at the probability of receiving a grant, aggregating single and multiple subsidized firms. To analyze potential differences in the selection process between single and multiple subsidized firms, we next split the sample running separate regression for each set of firms. This division is motivated by our expectation that multiple supported firms are more likely to engage in pure rent-seeking behavior, in the sense of being subsidy entrepreneurs, than are firms that merely received a single grant. To estimate the relationship between the categorical variable T_{it} and explanatory variables $X_{i,t-1}$, we use logistic regression. In all models, we choose to lag the time-dependent explanatory variables to avoid simultaneity bias.

In the dichotomous setting, we do not account for the number of grants received. With incomplete information about the cost of rent-seeking, single and multiple supported firms may deviate from each other. As a first step, we therefore continue and estimate an ordered logit model that can be considered an extension of the dichotomous logistic model. Specifically, we define three groups of firms: never, single and multiple subsidized firms. With this approach, we are take a first step in analyzing the shake-out that may occur as firms move one from being single to multiple subsidized.

As shown in Figure 2, the number of grants received by the multiple subsidized firms varies greatly. We thus ask if, already at the time of the first grant, it is possible to characterize the firm productivity, wages and skill intensity with respect to the number of grants that the firms will receive. That is, the count noted at time (t) is the maximum number of grants received by a firm. For the treated firms, we therefore delete observations beyond time t. With this setup, we also remove any potential treatment effects that may occur in future program participation.

To estimate the relationship between the count variables N_{it} and X_{it}, we turn to the multiplicative count data model. In our data, the number of zero counts widely surpasses the number of non-zero counts, which argues for a model that is capable of handling a large share of zero-valued observations. Here, we follow the suggestion of Silva and Tenreyro (2006, 2010), who argue for using the Poisson pseudo maximum likelihood (PPML) estimator over Poisson and negative binomial models. The advantages of the PPML-estimator include its ability to perform well in case of inflated zeros and its robustness against both under and over dispersion. The results are presented in the following section.

6 Results

Table 4: Logit regressions: Supported and non-supported firms.

	Full sample	Full sample
	(1)	(2)
Nr. employees	0.43***	0.46***
	(0.010)	(0.010)
Labor cost	0.26***	
	(0.030)	
Share high skill	0.013***	0.013***
	(0.000)	(0.000)
Equity ratio	-0.000	-0.000
	(0.000)	(0.000)
Labor productivity		-0.066***
		(0.020)
Constant	-8.54***	-6.84***
	(0.18)	(0.14)
Observations	1,889,491	1,858,065

Standard errors in parentheses

Dependent variable: Dummy variable for subsidized and non-subsidized firms. Only the first observation on subsidized firms is utilized. Cluster robust s.e. at firm level. Year, regional and industry fixed effects.

* p<0.1, ** p<0.05, *** p<0.01

Table 5: Logit regressions. Single and multiple supported firms.

	Single	Multiple	Single	Multiple
	(1)	(2)	(3)	(4)
Nr. employees	0.36***	0.61***	0.39***	0.64***
	(0.011)	(0.017)	(0.011)	(0.017)
Labor cost	0.23***	0.35***		
	(0.034)	(0.061)		
Share high skill	0.010***	0.021***	0.011***	0.021***
	(0.000)	(0.001)	(0.000)	(0.001)
Equity ratio	-0.000	0.000	-0.000	-0.000
	(0.000)	(0.000)	(0.000)	(0.000)
Labor productivity			-0.067***	-0.049
			(0.022)	(0.039)
Constant	-8.29***	-13.2***	-6.72***	-11.2***
	(0.20)	(0.53)	(0.16)	(0.49)
Observations	1,887,605	1,883,757	1,856,282	1,852,493

Standard errors in parentheses

Dependent variable: Dummy variable for subsidized and non-subsidized firms. Only the first observation on subsidized firms is utilized. Cluster robust s.e. at firm level. Year, regional and industry fixed effects.

* p<0.1, ** p<0.05, *** p<0.01

For supported firms, the results in table 4 suggest that productivity is negatively related to the probability of receiving a subsidy. This result suggests, in line with the theoretical model, that low-productivity firms have a low opportunity cost of seeking subsidies.

Seeking subsidies is not necessarily a simple process, and firms seek subsidies in competition with other firms. Hence, the ability to formulate and design an application can be instrumental to receiving a subsidy. It is therefore reasonable to assume that the ability to apply for a subsidy is positively correlated with the educational attainment of the labor force. In line with this argument, the results in table 4 suggest that subsidized firms have a relatively large share of workers with higher education. In addition, subsidized firms have relatively high salaries. Taken together, the results suggest that subsidized firms have low productivity, high salaries and a well-educated labor force.

In Table 5, we continue and separate multiple subsidized firms from single subsidized firms and compare each of these groups with non-subsidized firms. This operation reveals some interesting patterns. First, we note that multiple subsidized firms have a larger share of workers with tertiary education than do single subsidized firms, a property that signals a capacity of administrative matters, whereas there is a tendency of a productivity drop among

Table 6: Results from ordered logit regressions

	Full sample (1)	Full sample (2)
Nr. employees	0.44***	0.47***
	(0.010)	(0.010)
Labor cost	0.25***	
	(0.030)	
Share high skill	0.013***	0.013***
	(0.000)	(0.000)
Equity ratio	-0.000	-0.000
	(0.000)	(0.000)
Labor productivity		-0.066***
		(0.020)
Constant 1	26.3***	24.7
	(2.14)	(.)
Constant 2	27.7***	26.1
	(2.13)	(.)
Observations	1,995,787	1,963,450

Standard errors in parentheses

Dependent variable: Dummy variable indicating whether a firm receives zero, a single, or multiple supports. Only the first observation on subsidized firms is utilized. Cluster robust s.e. at firm level. Year, regional and industry fixed effects.

* p<0.1, ** p<0.05, *** p<0.01

Table 7: Results from PPML regressions.

	Full sample (1)	Full sample (2)
Nr. employees	0.42*** (0.074)	0.49*** (0.076)
Nr. employees squared	0.019 (0.012)	0.013 (0.012)
Labor cost	0.34*** (0.041)	
Share high skill	0.017*** (0.001)	0.017*** (0.001)
Equity ratio	0.000 (0.000)	-0.000 (0.000)
Labor productivity		-0.040 (0.027)
Constant	-26.8*** (0.21)	-24.9*** (0.17)
Observations	1,995,787	1,963,450

Standard errors in parentheses

Dependent variable: The maximum number of supports the firm will receive, measured at the first treatment. All post-treatment years are coded as missing. Non-subsidized firms receive a zero. Cluster robust s.e. at firm level. Year, regional and industry fixed effects.

* p<0.1, ** p<0.05, *** p<0.01

multiple subsidized firms, setting them below single and non-subsidized firms productivity-wise. Turning to the financial side, we note no large differences in the equity ratio between single and multiple subsidized firms, whereas multiple subsidized firms are larger than single subsidized firms (which, in turn, are larger than the average non-subsidized firm). Hence, the family of subsidized firms cannot be characterized as a population of the very smallest firms.

Finally, we note that after controlling for a series of firm characteristics, the average wage is higher in multiple subsidized firms than it is in single subsidized firms, which in turn is higher than that in non-subsidized firms.

The comparison between single and multiple subsidized firms is an interesting feature and central in the analysis of rent-seeking in our model. We therefore in Table 6 re-estimate the above model in an ordered logit model setting with the outcomes, non-, single, and multiple subsidized. Overall, the above results are maintained, but with one difference. Using an ordered logit, we see not only increasing wages and skill intensities as we go from single to multiple treated firms, but we also see significantly decreasing productivity measures as we move from single to multiple subsidized firms. Hence, the results are in line with the main prediction of our model, namely, that low-productivity firms are drawn to seeking grants. We also note that high-wage, skill-intensive firms tend to be over-represented among granted firms, indicating that seeking subsidies may require some administrative skills.

As noted above, the group of multiple treated firms is not entirely homogeneous: some firms receive two subsidies, but there is a set of firms receiving subsidies from more than 20 different programs. In our model, we have full information and no learning effects from applying grants; thus, one round is sufficient for separating rent-seekers from non-rent-seeking firms. One may, however, argue that if it is a learning process in rent-seeking and/or uncertainty about a firm's own rent-seeking ability, there can be a productivity shake-out as a firm goes from one grant to the next. For example, firms that experience a productivity increase above the threshold for program participation may opt out of further grant seeking.

An increasingly popular method of handling over-dispersion and zero-valued observations (here, non-subsidized firms) in a count data setting is given by the Pseudo Poisson maximum likelihood estimator (PPML). The advantages of the PPML estimator include its ability to perform well with a large fraction of zeros and its robustness against heteroscedasticity and under and over dispersion Silva and Tenreyro (2006, 2010). In Table 7, we present results that count the actual numbers of grants received by the firms and estimate the model using the PPML estimator. The count in Table 7 measures, at the time of the first grant (time (t)), the maximum number of grants a firm will receive during the period of observation. That is, a firm that is registered for four grants will be tagged with the number “four" at the time for the first grant. Thus, at the time of the first grant, we aim to characterize future multiple grant seekers.

The results from the PPML regressions show similar patterns as the initial logit regressions. To be precise, the results from the PPML estimations in Table 7 suggest that for each additional grant a firm will receive, at the time of the first grant, there is already a trend of higher wages, higher shares of skilled labor, and non-significant falling productivity measures accompanying the number of grants received.

Summing up, one may note that after comparing the results across various models, we find that both wages and firm skill intensity tend to increase with the number of grants received. For productivity, however, the results vary from a negative and significant trend to a negative and non-significant pattern.

6.1 Robustness checks

In this section, we analyze the robustness of the results with respect to the agency administrating the grants (Vinnova and SAERG), alternative productivity measures, firm size and removing firms with the highest number of received grants. The results are depicted in Table 8.

One potential argument against our results is that the aim and purpose of the programs vary across agencies. It might therefore be incorrect to lump them together. Given that Vinnova, to a larger extent than SAERG, aims to subsidize firms of high innovative capability, the firms that receive subsidies from Vinnova should respond better to these subsidies than should firms that receive subsidies from the regionalized SAERG subsidies. Separating firms with respect to the granting agency, we, in line with this hypothesis, find a difference across agencies. For SAERG (with a regional focus), we find negative productivity effects, but the opposite is true for Vinnova, where a positive productivity patterns is found. Turning to wages, we instead find negative wage effects from SAEREG, whereas the positive wage effect remains for Vinnova. However, when looking at skill intensity, the results are similar across agencies. [13] Moreover, one could question our choice of measurements of productivity. We therefore re-run the main regressions using productivity measurement of (Levinsohn and Petrin, 2003; Petrin et al., 2004) (henceforth referred to as LevPet). Changing the productivity measure does not upset the results with a positive relationship to wages and firm skill intensity. Productivity is, however, no longer significant.

Our last two robustness checks consider firm size and the number of received grants. There are a few firms that receive a relatively large number of grants and may thus drive the

[13]For firms receiving multiple subsidies from SEA, we have too few observations to analyze them separately.

results. As a robustness test, we remove firms that have received more than 25 grants from the analysis. Moreover, in many cases, the intention of the programs is to promote growth among SMEs. As a final robustness check, we therefore re-run the analysis on small firms only (firms with less than 50 employees). None of these robustness checks alter the main results. Subsidized firms can be characterized as having low productivity, high wages, and being skill intensive, and all of these characteristics become more pronounced as we move from single to multiple subsidized firms.

Table 8: Robustness checks

	Productivity	Wages	Skill-intensity
Agency - Vinnova	Positive***	Positive***	Positive***
Agency - SAERG	Negative***	Negative***	Positive***
LevPet Productivity	Negative	Positive***	Positive***
Less than 25 supports	Negative***	Positive***	Positive***
SME:s	Negative***	Positive***	Positive***

Note: Robustness checks, ordered logit estimates regressions.

* p<0.1, ** p<0.05, *** p<0.01

7 Conclusions

Solving market failures in the capital markets for innovative firms is a daunting task. Markets are generally good at aggregating information and allocating resources (Hayek, 1945). When markets fail, it is not obvious how governments should respond to solve this failure (Demsetz, 1969). On the one hand, governments might be able to correct the market failure and improve the equilibrium outcome with appropriately targeted subsidies and interventions. On the other hand, there is always a risk that subsidies are targeted toward the wrong firms or may trigger unintended behavior by distorting incentives.

In this paper, we aim to fill the gap in the research on the characteristics of firms drawn to selective and publicly sponsored, innovation and growth programs, a subject that has received too little attention (Zúñiga-Vicente et al., 2014).

To analyze which firms are drawn to selective innovation and growth-targeted programs, we develop a simple model of rent-seeking where the cost of seeking grants is matched by the cost of reallocating labor from productive work to rent-seeking. With this set-up, the alternative cost of seeking grants is higher for productive firms than it is for less productive firms. Hence, unproductive firms will self-select into rent-seeking activities. In this respect, we may note that without a proper understanding of which firms self-select into subsidies, it is more difficult to design an optimal policy.

We also hypothesize that firms might not fully discover their own capacity and the real costs of seeking grants until after they have participated in their first contest. Under this assumption, firms with a comparative advantage in rent-seeking will be further selected out after the first round. It therefore becomes interesting to split the sample of granted firms into non-, single, and multiple subsidized firms. Loosely speaking, we suggest that a low opportunity cost of seeking grants is consistent with low productivity in production and/or a high internal competence for designing applications. That is, there will be a shake-out where low productive firms continue to seek additional grants.

Using detailed firm-level data with information on the received grants and firms' input and output, we analyze how firm attributes, such as productivity, wages and skill intensity, influence a firm's likelihood of being a subsidized firm.

Overall, the results suggest that the firms receiving subsidies have low productivity, high salaries and a large share of workers with higher education. These characteristics are further pronounced when we separate multiple subsidized firms from single subsidized firms, although for productivity, this latter result is not fully robust with respect to the estimator used.

The tendency of weeding out high productivity as firms receive one additional grant can be a three-sided process. First, firms that experience high-productivity growth will opt out from future rent-seeking, leaving weaker firms to receive future grants. In addition, to the extent that there is a learning process involved in rent-seeking, low-productivity firms will allocate more resources to rent-seeking. At the same time, the agencies are most likely more interested in supporting promising firms rather than the weakest ones. Hence, there are selection forces working in both directions. The results, however, indicate that the impact of high-productivity firms leaving the rent-seeking contest and of a bias toward weaker firms specializing in rent-seeking dominates the productivity effect.[14]

A robust result found was that subsidized firms are more skill-intensive than are non-subsidized firms. This feature grows stronger with the number of grants received. Allowing for some speculation, writing comprehensive applications is a task that may require a special type of administrative skills in order to appear attractive. To the extent that such mechanisms are at work, firms with a large proportion of highly educated workers may have an advantage in the application process.

Finally, we found a drift toward higher wages among multiple subsidized firms. In our model, the wage setting is not further elaborated. A possible explanation for this result is

[14] In this study, we analyze the characteristics of supported firms rather than the strategies used by firms to win the contest. An analysis of the strategies used to win such contests could shed further light on this issue.

that the grants, to some extent, are treated as a windfall gain, allowing for a bonus when the grant is realized. This track is, however, not further elaborated here and is left for future research.

Some conclusions can be drawn based on these results: The evidence indicates that there is a certain population of firms that repeatedly receives public innovation and growth-targeted grants. Second, the population of supported firms can be characterized as having low productivity and high wages and being skill intensive. All of these characteristics become more pronounced as we move from single to multiple supported firms. That is, we cannot reject the hypothesis of the existence of a population of firms that, to some extent, specialize in rent-seeking. From a public policy perspective, these results suggest that the current way of supporting firms with innovation and growth-targeted grants can be problematic because it can create incentives for firms to specialize in unproductive entrepreneurship (Baumol, 1990).

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A Appendix

A.1 Further results

As an extension, we can also examine the response in expected profits from changes in the productivity term. The results are presented in the following proposition:

Proposition 4 (Comparative statics IV)**.** *For the j-th entrepreneur with $j \leq k$ and $k < n$ held fixed, we find that an increase in productivity A_j leads to*

(i) An positive (negative) change in the expected profits of entrepreneur j if the ratio between the efforts allocated to productive use and rent-seeking $(1 - h_j^*)/h_j^*$ is larger (smaller) than $2P(h_j^*) + 2k - 3$:

$$\frac{d\Pi_j\left(h_j^*\right)}{dA_j} = 1 - \frac{2\mu'(k-1)}{k\bar{A}_k} P(h_j^*)\left(1 - \frac{A_j}{k\bar{A}_k}\right) \gtrless 0, \quad \text{if } \frac{1 - h_j^*}{h_j^*} \gtrless 2\left(P(h_j^*) + k\right) - 3 \tag{26}$$

(ii) An increase in the expected profits of all other entrepreneurs gives

$$\frac{d\Pi_i\left(h_i^*\right)}{dA_j} = 1 + \frac{2\mu'}{k\bar{A}_k}\left(1 - P\left(h_i^*\right)\right) > 0 \tag{27}$$

Proof. To prove the inequality in (i), we use the fact that $1 - A_j/k\bar{A}_k = P(h_j^*) + k - 2$ along with (8) to rewrite the expression as

$$1 - 2h_j^*\left(P(h_j^*) + k - 2\right) > 0,$$

which can be expressed in terms of the ratio between the effort allocated to production and that which goes into rent-seeking:

$$\frac{1 - h_j^*}{h_j^*} > 2\left(P(h_j^*) + k\right) - 3.$$

In the case of *(ii)*, the inequality follows directly from $1 > P\left(h_j^*\right) > 0$ □

In the case of *(i)*, the result can be illustrated using Figure 1. The profits from production are in the area under the line from 1 to h_j^*, and the profits that result from rent-seeking are in the area under the dashed curve from 0 to h_j^*. A shock to productivity shifts both curves; the net effect on profits depends on the relative change in the increasing profits from production and decreasing profits from rent-seeking. However, when a shock to some other entrepreneur's productivity occurs, it affects only the profit function for the other rent-seeking contestants, resulting in a net increase in total profits. As the profits in production are totally determined by the productivity of entrepreneurs, a second implicit prediction is that low profits should correlate with entrepreneurs participating and, hence, winning the rent-seeking contest. Any such selection effect, however, should be counteracted with contests requiring the winning party to match the prize, so-called matching grants. However, these arguments are not pursued further in this paper.

We can also consider how h_j respond to an increase in the firm's own productivity A_j and to changes in another firm's productivity A_i for $i \neq j$. The results are as follows:

Proposition 5 (Comparative statics V). *For the j-th firm with $j \leq k$ where $k < n$ is fixed, we find that*

(i) A shock to the firm's productivity parameter A_j results in less allocation of h_j^*, provided $k \geq 2$, as given by the following partial derivative:

$$\frac{dh_j^*}{dA_j} = -\frac{\mu'(k-1)}{\left(k\bar{A}_k\right)^2}\left(2P\left(h_j^*\right) + k - 2\right) < 0, \quad \text{if } k \geq 2, \tag{28}$$

(ii) A shock to the productivity parameter A_j of some other firm $i \neq j$ with $i < n$ will result in less allocation of h_i to rent-seeking, provided $P\left(h_i^*\right) > 0.5$, and more allocation of h_j to rent-seeking if $P\left(h_i^*\right) < 0.5$. Evaluating the partial derivative of h_i with respect to A_j, we obtain

$$\frac{dh_i^*}{dA_j} = -\frac{\mu'(k-1)}{2\left(k\bar{A}_k\right)^2}\left(P\left(h_i^*\right) - \frac{1}{2}\right) \lesseqgtr 0, \quad \text{if } P\left(h_i^*\right) \gtreqless \frac{1}{2} \tag{29}$$

Proof. The results follow directly from $1 > P\left(h_i^*\right) > 0$. □

Beginning with (i), the interpretation is that if a firm experiences a positive shock to its own productivity, the relative cost of the effort used in rent-seeking increases, which reduces its equilibrium allocation h_j^*. In the case of (ii), when another firm experiences a positive shock to its productivity, then if firm j has a sufficiently low cost for its h_j (low A_i), it is preferable to allocate more to rent-seeking. However, if firm j is endowed with high productivity, this will induce the firm to allocate less to rent-seeking.

Next, we prove a result for the participation constraint when the productivity A_i is equidistantly distributed, as in the example given in Table 1. The participation constraint, i.e., how many firms that choose to enter the contest, crucially depends on how many entrepreneurs there are seeking rents, that is the value of k and how A_i is distributed among the n firms. In this section, we prove three independent results: first, when firms' A_i are distributed equidistantly from one another; second, when A_i has a uniform distribution; and third, when A_i has a exponential distribution. for some constant Δ. Then, we can prove the following results.

Proposition 6. *If the distribution of productivity is given by $A_{i+1} = A_i + \Delta$, with $A_1 > 0$ and $\Delta > 0$ and with the participation constraint given by (12), then there exists an upper bound to k, given by $\bar{k}$, such that k is the largest integer for which $k < \bar{k}$, and where $\bar{k}$ is given by the larger root of the following quadratic equation:*

$$k^2 - 3k + 2\left(1 - \frac{A_1}{\Delta}\right) = 0, \tag{30}$$

which takes the form

$$\bar{k} = \frac{1}{2}\left(\sqrt{1 + 8\frac{A_1}{\Delta}} + 3\right). \tag{31}$$

Proof. If $A_{i+1} = A_i + \Delta$, we can write $A_k = A_1 + (k-1)\,\Delta$. Inserting this expression into the participation constraint (12) gives

$$A_1 + (k-1)\,\Delta < \frac{\sum_{i=1}^{k-1}\left[A_1 + (i-1)\,\Delta\right]}{k-2}. \tag{32}$$

Using the fact that

$$\sum_{i=1}^{k-1}(i-1) = \sum_{i=0}^{k-2} i = \frac{(k-2)\,(k-1)}{2}, \tag{33}$$

the inequality reduces to

$$(k-2)\,(A_1 + (k-1)\,\Delta) < (k-1)\,A_1 + (k-2)\,(k-1)\,\frac{\Delta}{2}, \tag{34}$$

which, after simplifying the term, can be written as a quadratic expression of k

$$k^2 - 3k + 2\left(1 - \frac{A_1}{\Delta}\right) < 0 \tag{35}$$

Finding the roots of this expression gives two roots, of which the upper root corresponds to $\bar{k}$. The participation constraint in (12) is then satisfied for any k such that $k < \bar{k}$. □